Burlington County Institute of Technology

Medford Campus

Westampton Campus

Calculus

Department: Mathematics
Credits: 5
Revised: August 2023
Board Approval Date: August, 2023

Course Description

Calculus is an honors course that provides challenging and in-depth study of advanced mathematics. This course is designed for students who have a strong background in Precalculus and Algebra 2 and are highly motivated to explore mathematics in greater depth.

Calculus is a branch of mathematics that deals with the study of rates of change and continuous change. This course is designed to provide students with an understanding of the fundamental concepts of calculus, including limits, derivatives, and integrals. The course will also explore the various applications of calculus in fields such as physics, engineering, economics, and more.

The course will begin with an introduction to limits and continuity, followed by the study of derivatives and their applications in finding rates of change, optimization, and curve sketching. Students will then move on to the study of integrals, including substitution, and its applications in finding areas and volumes.

Throughout the course, students will be expected to develop their problem-solving skills through a combination of theoretical concepts and practical applications. By the end of the course, students should have a solid understanding of the fundamental concepts of calculus and be able to apply them to solve real-world problems.

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Pacing Guide

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Curriculum Maps

Unit 1: Limits and Continuity

Desired Outcomes

Established Goals: NJSLS

- Understand the concept of a function and use function notation (F.IF.1, F.IF.2).
- Interpret functions that arise in applications in terms of the context (F.IF.4).
- Analyze functions using different representations (F.IF.7).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:

- Calculus is the mathematics of change.
- The tangent line problem is basic to calculus.
- Limits can be used to describe continuity, the derivative, and the integral: the ideas giving the foundation of calculus.
- Limits can be used to describe the behavior of functions in absolute value for large numbers.

Essential Questions:

- What is a limit?
- How do you find a limit with a table? A graph? Analytically?
- What does indeterminate form mean? Which techniques can we use to solve these types of problems?
- When does a limit not exist?

○ Continuous functions are used to describe how a body moves through space and how the speed of a chemical reaction changes with time. ○ The tangent line determines the direction of a body's motion at every point along its path.	○ What is the difference between a one-sided and a two-sided limit? ○ How can limits be used to identify vertical and horizontal asymptotes? ○ How can limits be used to determine if a function is continuous? ○ Can all discontinuities be removed to create a continuous extension of the function? ○ What does the Intermediate Value Theorem allow us to find for continuous functions? ○ Why can we use our knowledge of average and instantaneous rates of change to find the slopes of secant and tangent lines to a curve?
Students will know: ○ Limits allow us to "straighten" curved lines and enable us to use algebra and geometry to solve real world problems. ○ Existence of a limit of a function, including at points, and as x approaches + or – infinity ○ Basic limit theorems. ○ One of the uses of limits is to test continuous functions which arise frequently in scientific work. ○ The definition of continuity as it applies to functions. ○ Types and examples of discontinuity. ○ Key Terms: secant line, tangent line, average speed, instantaneous speed, limit, polynomial function, rational function, limit, one-sided limit, indeterminate form, horizontal asymptote, vertical asymptote, end behavior model, continuity, discontinuity (removable and nonremovable),	**Students will be able to:** ○ Estimate a limit using a numerical or graphical approach. ○ Identify different types of behavior associated with nonexistence of a limit. ○ Evaluate a limit using properties of limits (scalar multiple, sum or difference, product, quotient, power). ○ Develop and use a strategy for finding limits. ○ Evaluate a limit using dividing out and rationalizing techniques. ○ Evaluate trigonometric limits using two special trigonometric limits. ○ Determine continuity at a point and continuity on an open interval. ○ Determine one-sided limits and continuity on a closed interval. ○ Use properties of continuity.

jump discontinuity, infinite discontinuity, oscillating discontinuity, continuous extension, greatest integer function, Intermediate Value Theorem, existence theorems	○ Understand and apply the Intermediate Value Theorem. ○ Determine infinite limits from the left and from the right. ○ Find and sketch the vertical asymptotes of the graph of a function. ○ Determine (finite) limits at infinity. ○ Determine the horizontal asymptotes, if any, of the graph of a function. ○ Determine infinite limits at infinity.

Assessment Evidence

Suggested Performance Tasks:

- Tangent Line Investigation
- 'Limits' Project using Excel spreadsheet
- Vertical Asymptotes Investigation using graphing utility

Required District/State Assessments:

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio

	○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

- Starter exercises
- Guided notes
- G.N.A.W. Assignments
- Mini-Assignments
- Formative assessments (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

- Science Connection: NJSLS HS-PS2-1
  - ⇒ Example: To escape the earth's gravitational field, a rocket must be launched with an initial velocity called the escape velocity. A rocket launched from the surface of the Earth has velocity in v (in miles per second) given by:

    $v = \sqrt{\frac{2GM}{r} + {v_0}^2 - \frac{2GM}{R}} \approx \sqrt{\frac{192{,}000}{r} + {v_0}^2 - 48}$

    where v_0 is the initial velocity, r is the distance from the rocket to the center of Earth, G is the gravitational constant, M is the mass of Earth, and R is the radius of Earth (approximately 4000 miles).
    - Find the value of v_0 for which you obtain an infinite limit for r as v tends to zero. This value of v_0 is the escape velocity for Earth.
    - A rocket launched from the surface of the moon has velocity v (in miles per second) given by

      $v = \sqrt{\frac{1920}{r} + {v_0}^2 - 2.17}$

Find the escape velocity for the moon.

- A rocket launched from the surface of a planet has velocity v (in miles per second) given by $v = \sqrt{\frac{10{,}600}{r} + v_0^{\,2} - 6.99}$
  Find the escape velocity for this plant. Is the mass of this planet larger or smaller than that of Earth? (Assume that the mean density of this planet is the same as that of Earth.)

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.
  - ⇒ Example: Students will be able to successfully troubleshoot complex problems that involve multiple approaches including research, analysis, reflection, interaction with peers, and drawing on past experiences.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
  - ⇒ Example: Engage in collaborative problem-solving activities that involve discussing and justifying solutions related to limits and continuity, presenting arguments for the existence or non-existence of limits, and explaining the concept of continuity to peers.

NJ SEL Competencies

- Self-Awareness: Recognize the impact of one's feelings and thoughts on one's own behavior
- Self-Management: Understand and practice strategies for managing one's own emotions, thoughts and behaviors

Culturally Relevant Connections

- *Present New Concepts Using Student Vocabulary:* Use student diction to capture attention and build

understanding before using academic terms.
    - ⇒ Example: Creating a scavenger hunt where students work together in groups to find new terms and their definition. http://www.classtools.net/QR/
- *Everyone has a Voice:* Create a classroom environment where students know that their contributions are expected and valued.
    - ⇒ Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- *Run Problem Based Learning Scenarios:* Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community.
    - ⇒ Example: Using a Place Based Education (PBE) model, students explore math concepts such as average and instantaneous speed while determining ways to address problems that are pertinent to their neighborhood, school or culture.
- *Research and Information Fluency:* Effectively use a variety of search tools and filters in professional public databases to find information to solve a real world problem.
    - ⇒ Example: Students can search through Paul's Online Math Notes, Khan Academy and other interactive sites for appropriate instructional videos and/or information pertaining to strategies and modeling of continuity and how it relates to limits.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies

ELL:

- Translation dictionary
- Sentence stems to provide additional language support for ELL students
- Pre-teach pertinent vocabulary
- Word wall with visual representations of mathematical terms
- Graphic organizers to help students interpret the meaning in an expression or equation in context

- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data

Enrichment

- Extended learning goals:
  - ⇒ Challenge Problem: Let P(x,y) be a point on the parabola $y=x^2$ in the first quadrant. Consider the triangle△PAO formed by P, A(0,1), and the origins O(0,0), and the triangle△PBO formed by P, B(1,0), and the origin.
    a. Write is the perimeter of each triangle in terms of x.
    b. Let r(x) be the ratio of the perimeters of the two triangles, $r(x) = \Delta PAO \div \Delta PBO$. Complete the table.

x	4	2	1	0.1	0.01
Perimeter ΔPAO					
Perimeter ΔPBO					
r(x)					

c. Calculate $\lim_{x \to 0+} r(x)$

⇒ Limits & Continuity Desmos Investigation:
https://teacher.desmos.com/activitybuilder/custom/574de5cdab71b5085a2aad42

⇒ Water Tank http://media.collegeboard.com/digitalServices/pdf/ap/ap16_calculus_bc_q1.pdf

Unit 2: Differentiation

Desired Outcomes

Established Goals: NJSLS

- Understand the concept of a function and use function notation (F.IF.1, F.IF.2).
- Interpret functions that arise in applications in terms of the context (F.IF.4).
- Analyze functions using different representations (F.IF.7).

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Enduring Understandings:

- The derivative gives the value of the slope of the tangent line to a curve at a point.
- Graphs of differentiable functions can be approximated by their tangent lines at points where the derivative exists.
- Rules for differentiations help us find derivatives of functions analytically more efficiently.
- Derivatives give the rates at which things change in the world.

Essential Questions:

- How can you use the definition of derivatives to determine algebraic derivatives of functions?
- What is the connection between derivatives and the graph of a function?
- How can a left-handed and right-handed derivative exist at a point but the derivative not exist there?
- When does the derivative at a point fail to exist?
- Does differentiability imply continuity? Does

○ The Chain Rule allows us to solve real world examples of composite functions such as normal daily maximum temperature for a given location. ○ Logarithmic differentiation is a powerful tool that aids in solving complex non-logarithmic functions.	continuity imply differentiability? ○ How are derivatives used to describe instantaneous rates of change? ○ What is the difference between displacement, average velocity and instantaneous velocity? ○ How can derivatives be used to solve problems in everyday life? ○ When is implicit differentiation necessary?
Students will know: ○ The relationship between a function, the slope of the tangent line, instantaneous rate of change, and the first derivative. ○ The difference between a continuous and differentiable function. ○ When to apply various differentiation rules (power, product, quotient, chain, etc.). ○ The derivative of various functions using the limit method and differentiation rules. ○ The derivative of trigonometric, exponential, and logarithmic functions. ○ When and how to differentiate relations implicitly. ○ Key terms: derivative, differentiable function, differentiation, derivative notation, derivative graphs, differentiability, difference quotient, locally linear, composite function, numerical derivative, jerk, Intermediate Value Theorem for Derivatives	**Students will be able to:** ○ Find the slope of the tangent line to a curve at a point using the derivative. ○ Use the limit definition to find the derivative of polynomial, radical, and rational functions. ○ Understand the relationship between differentiability and continuity. ○ Find the derivative of a function using the Constant Rule, Power Rule, Constant Multiple Rule, and Sum and Difference Rules. ○ Find the derivative of the sine and cosine functions. ○ Use derivatives to find rates of change. ○ Find the derivative of a function using the Product Rule and Quotient Rule. ○ Find the derivative of a trigonometric function. ○ Find a higher-order derivative of a function. ○ Calculate and apply a position, velocity, and acceleration function in real world applications. ○ Find the derivative of a composite function using the Chain Rule and General Power Rule. ○ Simplify the derivative of a function using algebra. ○ Find the derivative of a trigonometric function

	using the Chain Rule. ○ Distinguish between functions written in implicit and explicit form. ○ Use implicit differentiation to find the derivative of a function. ○ Develop properties of the natural exponential function. ○ Differentiate natural exponential functions ○ Develop and use properties of the natural logarithmic function. ○ Find derivatives of functions involving the natural logarithmic function. ○ Define exponential functions that have bases other than e. ○ Differentiate exponential functions that have bases other than e. ○ Find the derivative of an inverse function. ○ Differentiate an inverse trigonometric function. ○ Find a related rate. ○ Use related rates to solve real-life problems.
Assessment Evidence	
<u>Suggested Performance Tasks:</u> ○ Tangent Line Investigation using spreadsheet application ○ 'Projectile' project using spreadsheet application ○ 'Related Rates Problem Poster' project	**<u>Required District/State Assessments:</u>** ○ Unit Assessment ○ SGO Assessments **<u>Suggested Formative/Summative Assessments:</u>** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks)

	○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

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Learning Activities:

- Starter exercises
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- Guided notes
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

- Science Connection: NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1
  - ⇒ Water Temperature
    http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_bc_q1.pdf

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
    - ⇒ Example: Employ spreadsheet software like Microsoft Excel or Google Sheets to create tables and graphs for functions and their derivatives. Students can input different values and observe the changes in the function and its derivative, helping them grasp the relationship between inputs, outputs, and rates of change.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
    - ⇒ Example: Help students understand the relevance and applications of calculus and differentiation in various careers. Highlight professions that utilize calculus, such as engineering, physics, economics, computer science, and data analysis. Discuss how differentiation plays a role in optimizing designs, modeling real-world phenomena, and making informed decisions.

NJ SEL Competencies

- Social Awareness: Recognize and identify the thoughts, feelings and perspectives of others
- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking skills

Climate Change

- Students will complete one of the math explorations from Sustainability Math (http://sustainabilitymath.org/calculus-materials/):
    - ⇒ Mauna Loa Yearly Average CO2: Polynomial Differentiation, Tangent Line Problem
    - ⇒ Global Average Temperature: Function Composition, Polynomial Differentiation, Tangent Line Problem.
    - ⇒ Global Marine Surface Methan: Polynomial Differentiation, Tangent Line Problem.
    - ⇒ Arctic Sea Ice:

Culturally Relevant Connections

- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Creating a scavenger hunt where students work together in groups to find new terms and their definition. http://www.classtools.net/QR/
- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
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Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of

ELL:

- Translation dictionary
- Sentence stems to provide additional language support for ELL students.

mathematical terms ○ Teacher modeling of thinking processes involved in solving, graphing, and writing equations ○ Introduce concepts embedded in real-life context to help students relate to the mathematics involved ○ Record formulas, processes, and mathematical rules in reference notebooks ○ Graphing calculator to assist with computations and graphing of trigonometric functions ○ Utilize technology through interactive sites to represent nonlinear data ○ Graphic organizers to help students interpret the meaning of terms in an expression or equation in context	

Enrichment

- Extended learning goals:
  - ⇒ Calculus Shoebox Project : Make up a RELATED RATE word problem and illustrate in it a shoebox, using small toys, cutouts, pipe cleaner figures, play dough, etc. Write a clearly written statement of the problem (separate from box as well as IN box). Develop an answer key for your problem; include labeled drawing, formulas, and complete solution.

Unit 3: Applications of Differentiation

Desired Outcomes

Established Goals: NJSLS

- Understand the concept of a function and use function notation (F.IF.2).
- Interpret functions that arise in applications in terms of the context (F.IF.4, F.IF.5).
- Build a function that models a relationship between two quantities (F.BF.1).
- Build new functions from existing functions (F.BF.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
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Enduring Understandings:

- The derivative has both theoretical and real life applications.
- Derivatives provide useful information about the behavior of a function and its graph including its absolute maximum/minimum, relative maximum/minimum, where it is increasing/decreasing, its concavity and point(s) of inflection.

Essential Questions:

- How can derivatives be used to solve real world problems?
- How is the derivative presented graphically, numerically, and analytically?
- What is a critical number and what can they tell us?
- How is the derivative of a function f' related to the function f?

○ Understanding the rate of change of a function allows you to predict future behavior.	○ What are the similarities and differences between f' and f graphically? ○ What is the second derivative and how is it related to f and f'? ○ What does a derivative tell us about a function? ○ How can the derivative be used to solve optimization and related rate problems? ○ How do rates of change relate in real-life situations?
Students will know: ○ First and second derivatives are used to determine for a given function the critical values, intervals of increase and decrease, relative maxima and minima, points of inflection, and intervals concave up and concave down. ○ Key Terms: Absolute (Global) Extrema, Relative (Local) Extrema, Critical Point, Mean Value Theorem, Increasing and Decreasing Functions, antiderivative, antidifferentiation, First Derivative Test, Second Derivative Test, Concavity, Point of Inflection, Optimization, Maximum Profit, Minimizing Average Cost, Linear Approximation, Linearization, Newton's Method, Differentials, Absolute, Relative and Percentage Change, Related Rates	**Students will be able to:** ○ Understand the definition of extrema of a function on an interval. ○ Understand the definition of relative extrema of a function on an open interval. ○ Find extrema on a closed interval. ○ Understand and use Rolle's Theorem. ○ Understand and use the Mean Value Theorem. ○ Determine intervals on which a function is increasing or decreasing. ○ Apply the First Derivative Test to find relative extrema of a function. ○ Determine intervals on which a function is concave upward or concave downward. ○ Find any points of inflection of the graph of a function. ○ Apply the Second Derivative Test to find relative extrema of a function. ○ Analyze and sketch the graph of a function. ○ Solve applied optimization problems (minimum and maximum). ○ Understand the concept of a tangent line

	approximation. ○ Approximate a zero of a function using Newton's Method.

Assessment Evidence

Suggested Performance Tasks:	**Required District/State Assessments:**
○ The "Optimal" Can Project ○ Create Your Own Polynomial Project ○ Horizontal Asymptotes Investigation using graphing utility ○ 'Newton's Method' project using spreadsheets	○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

- Starter exercises
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- Guided notes
- In class activities (QR codes, scavenger hunt, interactive exercises, exit ticket, etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

- Science Connection: NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1
  - ⇒ Water Temperature http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_bc_q1.pdf

Technology (NJSLS Computer Science and Design Thinking)

- 8.2.12.ED.3 Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis.
  - ⇒ Example: Students will be able to use mathematical modeling to discuss and compare products to maximize profit and minimize cost.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
  - ⇒ Example: Maximize a Box with a Top Project - Students will design a box from a specific piece of poster board. They must maximize the volume of the box considering the size of the piece of board they are given.

NJ SEL Competencies

- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking skills

- Relationship Skills: Establish and maintain healthy relationships

Climate Change

- Students will complete one of the math explorations from Sustainability Math (http://sustainabilitymath.org/calculus-materials/):
  - ⇒ Arctic Sea Ice: Sixth degree polynomial differentiation, max/min, finding roots.
  - ⇒ Wind Energy by Selected Countries and World: Polynomial and logistic Differentiation, Quotient (or Product) Rule.
  - ⇒ U.S. Coal Consumption and Production: Fourth degree polynomial differentiation, max, finding roots.

Culturally Relevant Connections

- Call on Each Student: Encourage each student to share his or her thoughts through call-and-response, keeping the class's attention in the process.
  - ⇒ Example: Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, avoid handing out quizzes that are purely multiple choice or fill-in-the-blank. Among other question types, mix in problems that involve explaining the step necessary to get to the answer. After, give students time to assess their own progress and performance, helping them focus on growth.
- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures.
  - ⇒ Example: When learning about derivatives grab students' attention by including optimization problems that are relevant to their interest.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms.
  - ⇒ Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Gamify Lessons: Appeal to gaming culture by, for example, writing instruction manuals for projects and offering rewards such as badges.
  - ⇒ Example: Appeal to students' personal interest to keep them engaged. Create a game to check

students' understanding of applying the rules of derivatives.

⇒ https://kahoot.com/

⇒ http://www.classtools.net/

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context

ELL:

- Translation dictionary
- Sentence stems to provide additional language support for ELL students.

Enrichment
○ Students will complete related differentiation problems from previous AP exams: ⇒ chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-ab-practice-exam-2012.pdf

Unit 4: Integration

Desired Outcomes

Established Goals: NJSLS

- Understand the concept of a function and use function notation (F.IF.2, F.IF.3).
- Build a function that models a relationship between two quantities (F.BF.1, F.BF.2).
- Interpret expressions for functions in terms of the situation they model (F.LE.5).

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Enduring Understandings:

- Estimating with finite sums sets the foundation for understanding integral calculus.
- The definite integral is the basis of integral calculus, just as the derivative is the basis of differential calculus.
- The Fundamental Theorem of Calculus is the connection between derivatives and definite integrals.
- Definite integrals can be found using rectangular

Essential Questions:

- What is an integral?
- What is the connection between derivatives and integrals?
- How can we estimate the area under a curve using geometric shapes?
- What is the relationship between limits and areas under curves?
- How are the properties of definite integrals related to the Riemann Sum?

and trapezoidal numerical approximations.	○ How are definite integrals and areas related? ○ How would we define the average value of an arbitrary function f over a closed interval? ○ What is the Fundamental Theorem of Calculus?
Students will know: ○ The definite integral can be estimated using areas of plane regions as sums of rectangles constructed by using a partitioning of an interval and the right, left, midpoint or any point of the partition. ○ The definition of a definite integral is a limit to an infinite Riemann Sum, the exact area of the plane region. ○ The First and Second Fundamental Theorem applications in areas of simple plane regions, accumulation problems, and average-value-of-a-function problems. ○ Key Terms: Rectangular Approximation Method (RAM), LRAM, RRAM, MRAM, Riemann Sums, Sigma Notation, Definite Integral, Summation Notation, Integration Notation, Average Value of a Function, Mean Value Theorem for Definite Integrals, Antidifferentiation, Antiderivatives, Fundamental Theorem of Calculus, Trapezoidal Approximation	**Students will be able to:** ○ Write the general solution of a differential equation. ○ Use indefinite integral notation for antiderivatives. ○ Use basic integration rules to find antiderivatives. ○ Apply integration techniques to solve vertical motion problems. ○ Use sigma notation to write and evaluate a sum. ○ Approximate the area of a plane region (Riemann Sums and Trapezoidal Rule). ○ Find the area of a plane region using limits. ○ Evaluate a definite integral using limits. ○ Evaluate a definite integral using properties of definite integrals. ○ Evaluate a definite integral using the Fundamental Theorem of Calculus. ○ Understand and use the Mean Value Theorem for Integrals. ○ Find the average value of a function over a close interval. ○ Understand and use the Second Fundamental Theorem of Calculus. ○ Understand and use the Net Change Theorem. ○ Use the General Power Rule for Integration to find an indefinite integral. ○ Evaluate a definite integral involving an even or odd function.

	○ Use a change of variables to evaluate an indefinite and definite integral (u-substitution). ○ Use the Log Rule for Integration to integrate a rational function. ○ Integrate trigonometric functions. ○ Integrate natural exponential functions. ○ Integrate functions whose antiderivatives involve inverse trigonometric functions.

Assessment Evidence

Suggested Performance Tasks:

- A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second.
  a) How long will it take the ball to rise to its maximum height?
  b) What is the maximum height?
  c) When is the velocity of the ball one-half the initial velocity?
  d) What is the height of the ball when its velocity is one-half the initial velocity?

- An experimental vehicle is tested on a straight track. It starts from rest, and its velocity *v* (meters per second) is recorded in the table every 10 seconds for 1 minute.

t	0	10	20	30	40	50	60
v	0	5	21	40	62	78	83

  a) Use a graphing utility to find a model of the

Required District/State Assessments:

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
  Observation
- Graphic Organizers/ Concept Mapping

form $v = at^3 + bt^2 + ct + d$ for the data. b) Use a graphing utility to plot the data and graph the model. c) Use the Fundamental Theorem of Calculus to approximate the distance traveled by the vehicle during the test.	○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

- Science Connection: NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1
  - ⇒ Metal Wire (pg. 4) http://apcentral.collegeboard.com/apc/public/repository/_ap05_sg_calculus_ab_46569.pdf

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
  - ⇒ Example: Students will explore the application of integration concepts by performing numerical approximations of definite integrals. Through the use of Google Spreadsheets, students will set up a spreadsheet, input function values, and calculate the areas of rectangles formed using the function and interval widths. They will analyze the results, compare with known values, and discuss the concept of error and factors affecting accuracy. The lesson encourages students to experiment, reflect on the

benefits and limitations of using Google Spreadsheets for numerical approximations, and offers extension activities for further exploration.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
  - ⇒ Example: Engage in collaborative problem-solving activities that involve discussing and explaining solutions to integration problems, presenting integration techniques, and justifying the steps involved in solving integrals.

NJ SEL Competencies

- Self-Awareness: Recognize the impact of one's feelings and thoughts on one's own behavior
- Self-Management: Recognize the skills needed to establish and achieve personal and educational goals

Climate Change

- Students will complete one of the math explorations from Sustainability Math (http://sustainabilitymath.org/calculus-materials/):
  - ⇒ Ecosystem Service - Rain Runoff: Understand the ecosystem service provided by trees and shrubs in absorbing water runoff.
  - ⇒ US and China Co2 Emissions: The use of Riemann sums is basic, but the overall assignment is rich in context and demonstrates the value of area under a curve.
  - ⇒ World Petroleum Consumption: Setting up and solving and equation with an integral and one with just a function.
  - ⇒ Gini Coefficient Transition to Integration: Overall, this assignment moves from differentiation ideas with an application of the mean value theorem to integration. Once students can integrate, you can return to this project and have students find the Gini coefficients. Note that you will need some form of technology to solve higher order polynomials for zero. This project is a really nice way to motivate the need for the area under the curve before starting an integration unit.

Culturally Relevant Connections

- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Work with students to create a variety of sorting and match games of vocabulary words in this unit. Students can work in teams or individually to play these games for approximately 10-15 minutes each week. This will give students a different way of becoming familiar with the vocabulary rather than just looking up the words or writing the definition down.
- Use Media that Positively Depict a Range of Culture: Include different cultures and languages in your curriculum by presenting relevant material, such as movies, about them. Example: Use multiple approach such as an online component that can be shared with students and parents. Work as a facilitator and set a timeline for students to accomplish tasks. https://quizlet.com/subject/integrals/

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical

ELL:

- Translation dictionary
- Sentence stems to provide additional language support for ELL students.

rules in reference notebooks ○ Graphing calculator to assist with computations and graphing of trigonometric functions ○ Utilize technology through interactive sites to represent nonlinear data ○ Graphic organizers to help students interpret the meaning of terms in an expression or equation in context	
Enrichment	
○ Students will complete related integral problems from previous AP exams: ⇒ chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-ab-practice-exam-2012.pdf	

Unit 5: Applications of the Definite Integral (Supplemental)

Desired Outcomes

Established Goals: NJSLS

- Understand the concept of a function and use function notation (F.IF.2).
- Interpret functions that arise in applications in terms of the context (F.IF.4, F.IF.5).
- Build a function that models a relationship between two quantities (F.BF.1).
- Build new functions from existing functions (F.BF.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

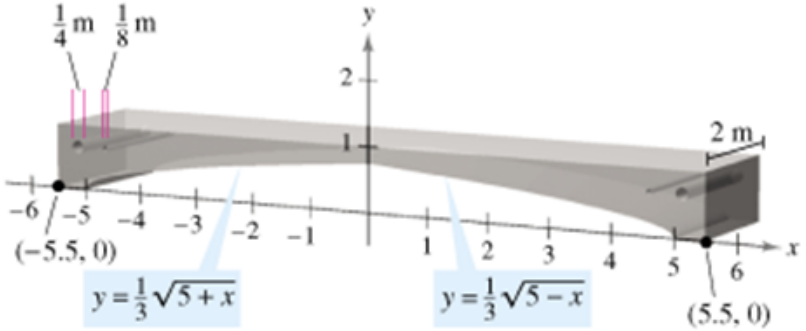
Enduring Understandings:

- The integral is a tool that can be used to calculate net change and total accumulation.
- Integration techniques allow us to compute areas of complex regions of the plane.
- The volume of a variety of three dimensional solids can be obtained using integration.

Essential Questions:

- How can integration be used to find the area between two curves?
- How can integrals be applied to finding a generated volume?
- How can integrals be used to find volumes of complex figures?
- What are the practical applications of finding the volumes of complex figures?

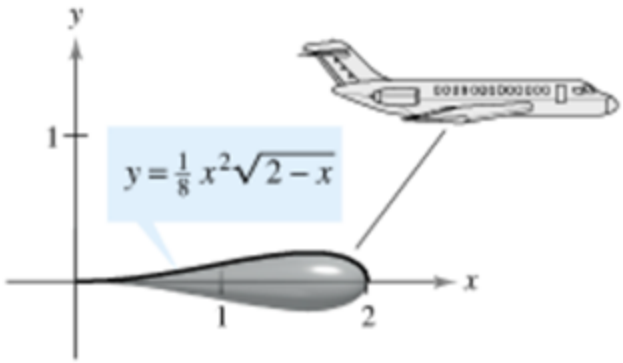
	○ How does the graph effect the way that area/volume is determined?
Students will know: ○ The definite integral is used to find the areas of regions between curves using all types of functions. ○ The definite integral is used to find the volume of a region rotated around an axis. ○ Key Terms – Area Between Curves, Area Enclosed By Intersecting Curves, Volume of a Solid with known Cross Sectional Area, Disk Method, Washer Method	**Students will be able to:** ○ Find the area of a region between two curves using integration. ○ Find the area of a region between intersecting curves integration. ○ Find the volume of a solid of revolution using the disk method, washer method, and with known cross sections. ○ Find the volume of a solid of revolution using the shell method.
Assessment Evidence	
Suggested Performance Tasks: ○ Concrete sections for a new building have the dimension (in meters) and shape shown in the figure.	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items



- Find the area of the face of the section superimposed on the rectangular coordinate system.
- Find the volume of concrete in one of the sections by multiplying the area in part (a) by 2 meters.
- One cubic meter of concrete weights 5000 pounds. Find the weight of the section.

- A tank on the wind of a jet aircraft is formed by revolving the region bounded by the graph of y=1/8 x^2 √(2-x) and the x-axis about the x-axis (see figure), where x and y are measured in meters. Find the tank's volume.

- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing



Learning Plan

Learning Activities:

- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

- Science Connection: NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1
  - ⇒ Metal Wire http://apcentral.collegeboard.com/apc/public/repository/_ap05_sg_calculus_ab_46569.pdf

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena
  - ⇒ Example: In this application project, students use spreadsheets to model and analyze population

growth using definite integrals in calculus. By importing historical population data, students calculate rates of change and approximate total population growth within specific time intervals using numerical integration methods. They visualize the data, rates of change, and approximated growth using line charts or scatter plots, and analyze the trends and patterns observed. This project enhances students' skills in data analysis, numerical approximation, and utilizing computational tools to solve real-world problems in calculus, specifically related to population dynamics.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.1.12.RM.1: Describe the importance of various sources of income in retirement, including Social Security, employer-sponsored retirement savings plans, and personal investments.
  - ⇒ Example: Students will apply the integral to investment and depreciation.

NJ SEL Competencies

- Social Awareness: Demonstrate an understanding of the need for mutual respect when viewpoints differ
- Responsible Decision-Making: Identify the consequences associated with one's actions in order to make constructive choices

Culturally Relevant Connections

- Call on Each Student: Encourage each student to share his or her thoughts through call-and-response, keeping the class's attention in the process. Example: Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, avoid handing out quizzes that are purely multiple choice or fill-in-the-blank. Among other question types, mix in problems that involve explaining the step necessary to get to the answer. After, give students time to assess their own progress and performance, helping them focus on growth.
- Encourage Students to Propose Ideas for Projects: Let students take projects from concept to completion by pitching you idea, allowing then to showcase their strengths. Example: Students will take accountability to develop project ideas that meet academic standards. You will act as a coach to guide students that falls short to refine their ideas, if he/or she can't refine their ideas to meet the standards they can choose from a project list of options you provide. https://www.maplesoft.com/applications/category.aspx?CID=156

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context

ELL:

- Translation dictionary
- Sentence stems to provide additional language support for ELL students.

Enrichment

- Extended learning goals:

⇒ Encourage Students to Propose Ideas for Projects: Let students take projects from concept to completion by pitching your idea, allowing them to showcase their strengths. Example: Students will take accountability to develop project ideas that meet academic standards. You will act as a coach to guide students that fall short to refine their ideas, if he/or she can't refine their ideas to meet the standards they can choose from a project list of options you provide.
https://www.maplesoft.com/applications/category.aspx?CID=156

Appendix A: Culturally Relevant Pedagogy Examples

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

CONTENT INTEGRATION	KNOWLEDGE CONSTRUCTION	PREJUDICE REDUCTION	EQUITABLE PEDAGOGY	EMPOWERING SCHOOL CULTURE
Teachers use examples and content from a variety of cultures & groups.	Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.	Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.	Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.	Using the other four dimensions to create a safe and healthy educational environment for all.
This unit / lesson is connected to other topics explored with students. There are multiple viewpoints reflected in the content of this unit / lesson. The materials and resources are reflective of the diverse identities and experiences of students. The content affirms students, as well as exposes them to experiences other than their own.	This unit / lesson provides context to the history of privilege and oppression. This unit / lesson addresses power relationships. This unit / lesson help students to develop research and critical thinking skills. This curriculum creates windows and mirrors* for students.	This unit / lesson help students question and unpack biases & stereotypes. This unit / lesson help students examine, research and question information and sources. The curriculum encourage discussion and understanding about the groups of people being represented. This unit / lesson challenges dominant perspectives.	The instruction has been modified to meet the needs of each student. Students feel respected and their cultural identities are valued. Additional supports have been provided for students to become successful and independent learners. Opportunities are provided for student to reflect on their learning and provide feedback.	There are opportunities for students to connect with the community. My classroom is welcoming and supportive for all students? I am aware of and sensitive to the needs of my students and their families. There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Developed by Karla E. Vigil. Adapted with permission from James A. Banks, CULTURAL DIVERSITY AND EDUCATION: FOUNDATIONS, CURRICULUM, AND TEACHING (6th edition), New York: Routledge, 2016, page 5 and Gordon School Institute on Multicultural Practice.

Appendix B: English Language Learners

WIDA Levels:

At the given level of English language proficiency, English language learners will process, understand, produce or use

6- Reaching	• Specialized or technical language reflective of the content areas at grade level • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • Oral or written communication in English comparable to proficient English peers
5- Bridging	• Specialized or technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports • Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.
4- Expanding	• Specific and some technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs • Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support
3- Developing	• General and some specific language of the content areas • Expanded sentences in oral interaction or written paragraphs • Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support
2- Beginning	• General language related to the content area • Phrases or short sentences • Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support
1- Entering	• Pictorial or graphic representation of the language of the content areas • Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*	Graphic Supports*	Interactive Supports*	Verbal and Textual Supports
• Real-life objects (realia) or concrete objects • Physical models • Manipulatives • Pictures & photographs • Visual representations or models such as diagrams or drawings • Videos & films • Newspapers or magazines • Gestures • Physical movements • Music & songs	• Graphs • Charts • Timelines • Number lines • Graphic organizers • Graphing paper	• In a whole group • In a small group • With a partner such as *Turn-and-Talk* • In pairs as a group (first, two pairs work independently, then they form a group of four) • In triads • Cooperative learning structures such as *Think-Pair-Share* • Interactive websites or software • With a mentor or coach	• Labeling • Students' native language • Modeling • Repetitions • Paraphrasing • Summarizing • Guiding questions • Clarifying questions • Probing questions • Leveled questions such as *What? When? Where? How? Why?* • Questioning prompts & cues • Word Banks • Sentence starters • Sentence frames • Discussion frames • Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

Appendix C: WIDA ELD Standards Integration

ELD-MA.9-12 Explain Interpretive	Interpret mathematical explanations by • Identifying concept or entity • Analyzing data and owning problem-solving approaches • Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles
ELD-MA 9-12 Explain Expressive	Construct mathematical explanations that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions
ELD-MA.9-12 Argue Interpretive	Interpret mathematics arguments by • Comparing conjectures with previously established results and stated assumptions • Distinguishing correct from flawed logic • Evaluating relationships among evidence and mathematical principles to create generalizations
ELD-MA.9-12 Argue Expressive	Construct mathematics arguments that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions

Appendix D: Differentiated Instruction

Strategies to accommodate based on student individual needs::

1. Time/General
   a. Extra time for assigned tasks
   b. Adjust length of assignment
   c. Timeline with due dates for reports and projects
   d. Communication system between home and school
   e. Provide lecture notes/outline

2. Processing
   a. Extra Response time
   b. Have students verbalize steps
   c. Repeat, clarify or reword directions
   d. Mini-breaks between tasks
   e. Provide a warning for transitions
   f. Partnering

3. Comprehension
   a. Precise processes for balanced math instructional model
   b. Short manageable tasks
   c. Brief and concrete directions
   d. Provide immediate feedback
   e. Small group instruction
   f. Emphasize multi-sensory learning

4. Recall
   a. Teacher-made checklist
   b. Use visual graphic organizers
   c. Reference resources to
   d. promote independence
   e. Visual and verbal reminders
   f. Graphic organizers

5. Assistive Technology
   a. Computer/whiteboard
   b. Tape recorder
   c. Video Tape

6. Tests/Quizzes/Grading
   a. Extended time
   b. Study guides
   c. Shortened tests
   d. Read directions aloud

7. Behavior/Attention
   a. Consistent daily structured routine
   b. Simple and clear classroom rules
   c. Frequent feedback

8. Organization
   a. Individual daily planner
   b. Display a written agenda
   c. Note-taking assistance
   d. Color code materials

Appendix E: Enrichment

What is the purpose of enrichment?

The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.

- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is ...	**Enrichment is not...**
• Planned and purposeful • Different, or differentiated, work – not just more work • Responsive to students’ needs and situations • A promotion of high-level thinking skills and making connections within content • The ability to apply different or multiple strategies to the content • The ability to synthesize concepts and make real world and cross curricular connections • Elevated contextual complexity • Sometimes independent activities, sometimes direct instruction • Inquiry based or open-ended assignments and projects • Using supplementary materials in addition to the normal range of resources • Choices for students • Tiered/Multi-level activities with flexible groups (may change daily or weekly)	• Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment) • Worksheets that are more of the same (busywork) • Random assignments, games, or puzzles not connected to the content areas or areas of student interest • Extra homework • A package that is the same for everyone • Thinking skills taught in isolation • Unstructured free time

Appendix F: Resources

Textbook: Calculus Eighth Edition; Larson, Hostetler, Edwards; Houghton-Mifflin; 2006

Appendix G: Climate Change Curriculum Statement

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. These standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

Districts are encouraged to utilize the NJSLS to develop interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented. While the 2016 NJSLS-English Language Arts (ELA) and Mathematics do not have specific climate change standards, districts may want to consider how they can design interdisciplinary climate change units that incorporate relevant ELA and mathematics standards.

Components of this are tagged throughout the curriculum as appropriate under the “Related Standards” section in each unit.